

Investigation on the Bending Characteristics of Variable Thickness Negative Stiffness Honeycomb Core

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ABSTRACT

This study investigates the bending behavior and energy absorption capacity of negative stiffness honeycomb cores with variable thickness under flexural loading. The objective is to improve structural performance by introducing local beam thickness modification in three models (LM1, LM2, and LM3) and by varying the initial beam height ($h = 2.8-9$ mm). Both experimental testing and finite element simulations using Abaqus CAE were conducted, with validation confirming good agreement between experimental and simulation results. Among the locally modified models, LM3 demonstrated the best overall performance, achieving the highest peak force and the greatest specific energy absorption (SEA). This corresponds to a 29 % increase in peak force and a 25.6 % improvement in SEA compared to the baseline constant-thickness core. Although LM3 showed sudden post-peak drops, its superior strength and energy dissipation make it well-suited for applications requiring high impact resistance. Furthermore, varying the initial beam height to 8 mm significantly enhanced the performance, delivering up to 95 % higher peak force and 142 % greater SEA. Overall, the results highlight the potential of functionally graded honeycomb cores for crash protection, aerospace, and vibration isolation systems.

Keywords: Honeycomb Structure, Double Curved Beam, Specific Energy Absorption, Peak Force, Negative Stiffness



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1. Introduction

Sandwich structures, made of thin face sheets bonded to a lightweight core, are widely used in aerospace, marine, and automotive applications for their high strength and stiffness-to-weight ratios [1], [2], [3], [4]. Core designs include foams, lattices, corrugated sheets, and honeycombs, selected based on performance demands. Honeycomb cores are particularly attractive due to their strength-to-weight efficiency, energy absorption, and low cost [5], [6]. Foam cores are attractive for their low density and energy absorption, but face limitations in scalability and customization [7]. Advances in architected metamaterials, where geometry dictates performance, have introduced tunable structures with superior energy absorption and recoverability [8]. Among them, negative stiffness (NS) metamaterials are notable for their bistable force-deflection response, snap-through buckling, and enhanced damping [9], [10], [11], [12]. Negative stiffness honeycombs (NSH), formed by curved beams, undergo snap-through under transverse loads [13], [14]. While enabling recoverable absorption, this also causes sudden load drops and instability [15], [16], [17]. Since snap-through is sensitive to imperfections, geometric modifications such as beam thickening or height variation are used to delay it, though often at the cost of reduced absorption [18]. This study investigates the bending behavior of variable-thickness negative stiffness honeycomb cores. Both experimental and numerical approaches are employed to evaluate energy absorption and structural stability. The primary objectives are to (i) examine energy absorption experimentally and numerically, (ii) optimize geometric parameters such as local beam thickness and initial height, and (iii) improve mechanical performance by mitigating

early snap-through while enhancing load capacity and energy absorption efficiency.

2. Literature & Theory

Honeycomb structures are valued for their lightweight design and energy absorption under impact, making them common in aerospace and protective systems [19]. Conventional honeycombs, however, deform plastically and irreversibly [20]. NS honeycombs overcome this through snap-through buckling of curved beams, which allows recoverable energy absorption and superior damping compared to foams [19], [20], [21]. As architected metamaterials, NSH exploit geometry to achieve features such as multi-stability, recoverability, and elastic buckling [8], [10]. Several modifications have been explored: composite beams improve stiffness [22], while gradient thickness honeycombs offer better vibration damping and plateau stresses under large deformations [13], [18] reported that functionally varied thickness raised energy absorption by 11% and shear strength by 33% over uniform designs. In theory, negative stiffness results from curved or pre-buckled beams that exhibit snap-through under load [13], [14]. This bistable mechanism provides high absorption but risks sudden instability and load loss [15], [16], [17]. Geometric optimization, therefore, is critical to balance delayed snap-through with sufficient absorption.

3. Background with related work

Lightweight sandwich cores, such as foams and constant-thickness honeycombs, have long been used for stiffness and energy absorption, but often fail due to irreversible plastic deformation [20], [21]. To address this,

researchers developed metamaterial-based cores where geometry governs performance. Negative stiffness honeycombs, composed of pre-buckled beams, are promising due to their snap-through response, enabling recoverable absorption and tunable mechanics [14], [19], [22]. Studies show that gradient thickness [13], [18] and composite beam designs [22] improve peak force, energy absorption, and vibration control. Despite these advances, mitigating sudden instability remains a challenge.

The present work extends this research by exploring variable-thickness and height-modified NS honeycombs under flexural loading. Combining experiments with simulations, it seeks to optimize geometric parameters for improved load capacity, delayed instability, and efficient energy absorption.

4. Methods

The methodology of this study followed a systematic approach combining experimental investigations and finite element analysis (FEA) to examine the bending behavior of variable-thickness negative stiffness honeycomb (NSH) cores. The process involved three major steps: model design, experimental setup, and numerical simulation.

4.1 Model Design

A baseline honeycomb model was developed to represent the negative stiffness configuration. Each unit cell used a double-curved beam (DCB) design, based on the functionally graded NS metamaterial by Aewzipo et al. The honeycomb consisted of six columns and three rows of DCB cells, linked with horizontal and vertical stiffeners to limit excessive rotation. The baseline constant-thickness core had a beam thickness of 0.85 mm, height of 2.8 mm, length of 32 mm, and cell width of 35 mm. CAD models for both the baseline and modified designs were created in SolidWorks 2022. Equation 1 defines the beam's shape, while Fig. 1 shows the CAD representations of the beam, unit cell, and the full 3D honeycomb core [18].

$$W(x) = \frac{h_r}{2} [\cos(2\pi \frac{x}{L}) - 1] \quad (1)$$

4.2 Experimental Setup

The honeycomb cores were 3D-printed using a Bambu Lab P1S with 100% infill density to ensure solid structural integrity. The cores were printed using PETG with a layer height of 0.2 mm and nozzle diameter of 0.4 mm. After printing, specimens were carefully trimmed and inspected to ensure dimensional accuracy and surface uniformity. Three point bending tests were conducted on a Shimadzu 300 kN Universal Testing Machine, using a 150 mm span length and displacement-controlled loading at 6 mm/min for 300 s. A 10 mm diameter cylindrical indenter applied the load, while two 30 mm diameter support cylinders provided boundary conditions. The setup, shown in Fig. 2, followed ASTM C393M guidelines for core bending tests. Force-displacement data were continuously recorded, and deformation sequences captured for qualitative comparison with simulations.

4.3 Finite Element Analysis

Numerical simulations were conducted in Abaqus CAE 6.14 to replicate the experimental setup (Fig. 3a). Honeycomb models were meshed with C3D8 (8-node hexahedral) elements and a refined 0.5 mm mesh, determined via convergence study (Fig. 3b).

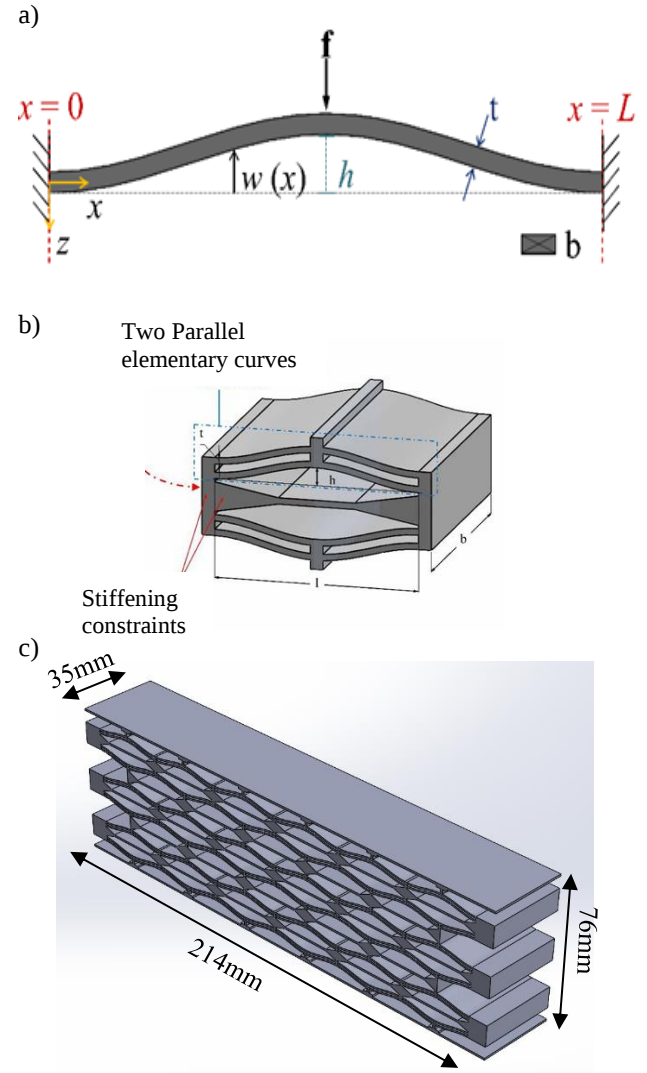


Fig. 1 CAD model of a) elementary curved beam, b) unit cell, and c) constant thickness DCB core

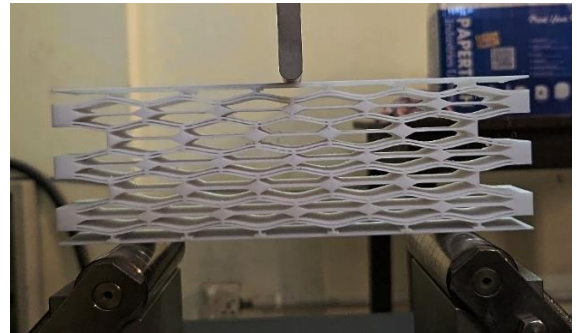


Fig. 2 Experimental setup for 3-point bending test of DCB honeycomb core

In the finite-element analysis (FEA) conducted in ABAQUS, the discretized equilibrium equation may be expressed in matrix form as $[K]\{u\} = \{F\}$, where $[K]$ is the global stiffness matrix, $\{u\}$ is the global nodal displacement vector, and $\{F\}$ is the global load vector. The compact algebraic form arises from the weak-form (principle of virtual work) of the governing differential equations $L\phi + f = 0$ and boundary conditions $B\phi + g = 0$. [23] Displacement-controlled loading of 30 mm was applied, with surface-to-surface contact and a friction coefficient of 0.2. The PETG material was modeled as elastic-plastic, with a Young's

modulus of 738 MPa, Poisson's ratio of 0.34, and yield strength of 43 MPa. Advanced contact algorithms ensured accurate simulation of snap-through behavior.

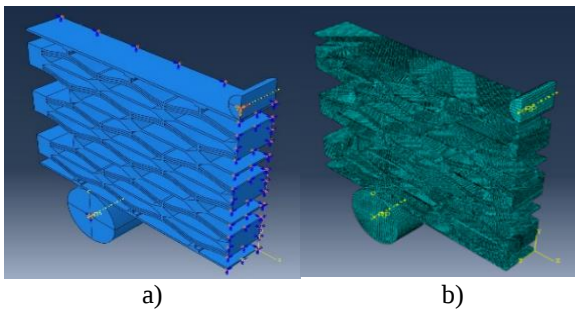


Fig. 3 a) Loading condition, and b) meshing in Abaqus CAE for the 3-point bending test of constant thickness honeycomb core

5. Results

The results are presented in two parts: validation of the numerical model and evaluation of structural modifications.

5.1 Model Validation

Comparison of experimental and numerical results for the constant-thickness baseline showed good agreement (Fig. 4). Both captured the elastic response up to 10 mm displacement, followed by localized buckling at 15-20 mm, and load drops from snap-through.

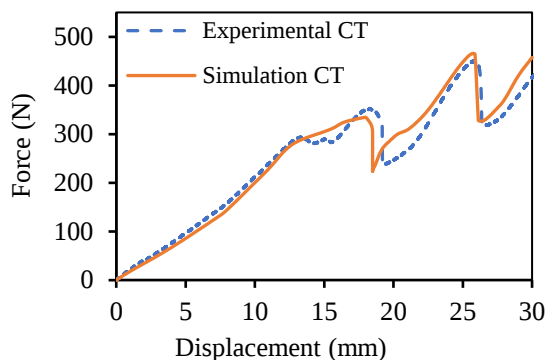


Fig. 4 Validation of simulation with the experiment of a constant thickness honeycomb core

Experimental curves exhibited oscillations due to imperfections, while simulations were smoother. Despite minor differences, critical loads and overall deformation patterns matched closely, confirming the FEA model's reliability.

5.2 Effect of Local Thickness Modification

Local beam thickening significantly improved flexural performance. From Figs. 5 to 7, beams are highlighted that experienced snap-through buckling. Hence, the thickness of the local beams was increased to 1 mm from 0.85 mm to prevent snap-through buckling by enhancing the stiffness of the beams.

In Fig. 8, it is observed that the local thickening of honeycomb beams improves strength and stability compared to the constant-thickness (CT) model. LM1 increased the peak load but still exhibited snap-through. LM2 provided the smoothest and most stable response, with better energy absorption, and LM3 achieved the highest load capacity but suffered from brittle-like drops. Overall, LM2 provided the best balance between strength and stability. From Fig. 9, it is

observed that CT has the lowest performance (465 N, 1.95 mJm³/kg). LM1 improved to 502 N (+8%) and 2.15 mJm³/kg (+10.3%).

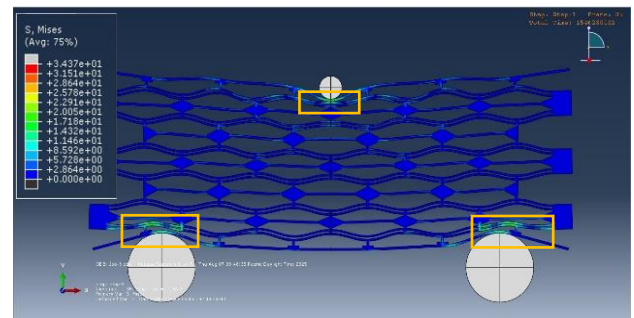


Fig. 5 Beams highlighted for increasing thickness for Local Modification-1 (LM1)

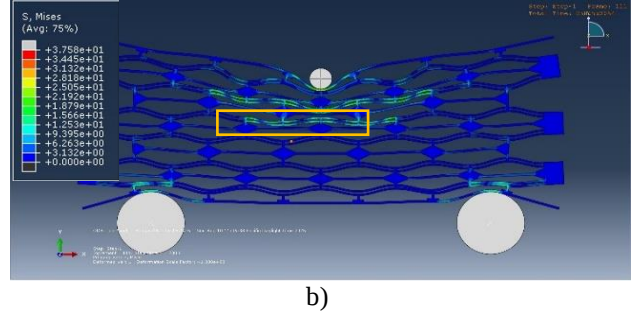
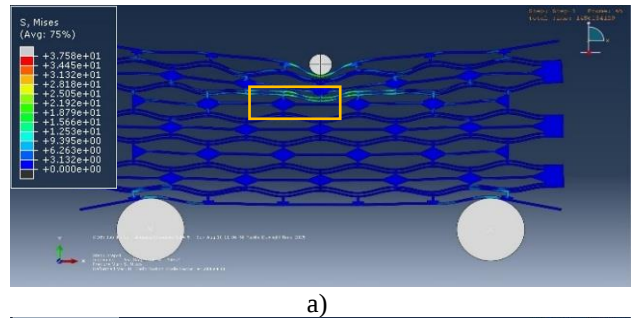


Fig. 6 Beams highlighted for increasing thickness for LM2

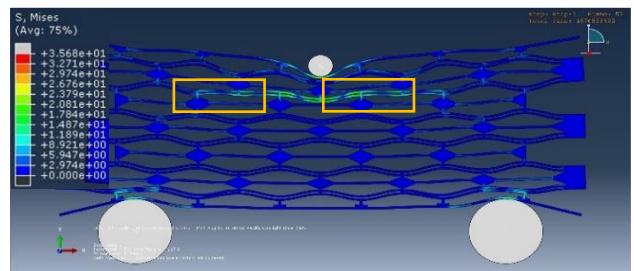


Fig. 7 Beams highlighted for increasing thickness for LM3

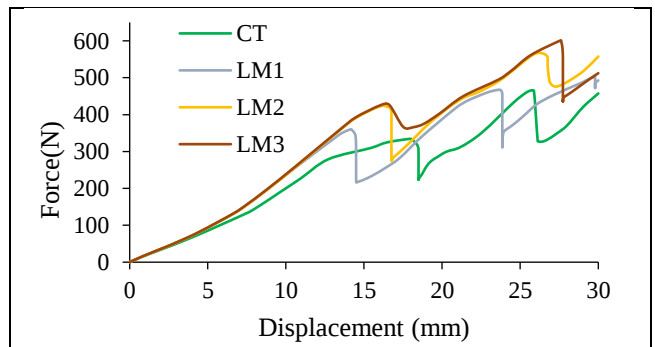


Fig. 8 Comparison of the force vs displacement curve of increased local beam thickness

LM2 achieved 564 N (+21.3%) and the highest SEA of 2.41 mJm³/kg (+23.6%), making it optimal. LM3 reached the highest load (600 N, +29%) but only a slight SEA gain (2.45 mJm³/kg, 1.7% over LM2).

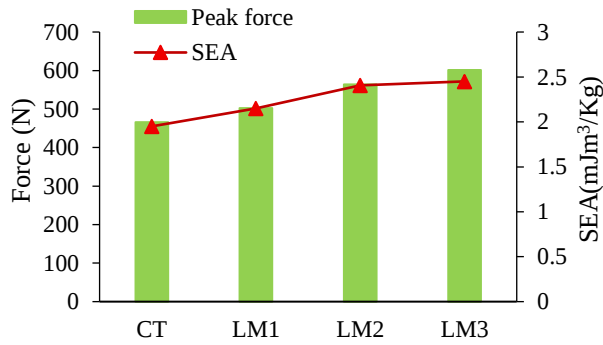


Fig. 9 Comparison of peak force and Specific Energy Absorption of differential beam thickness models

5.3 Effect of Initial Beam Height

Figs. 10 and 11 exhibit that the flexural performance of honeycomb beams improved notably with increasing beam height. At the lowest height of $h = 2.8$ mm, the structure showed poor behavior, with a peak force of about 350 N and the lowest SEA of 1.95 mJm³/kg. Increasing the height to $h = 4$ mm enhanced the peak force to 500 N (+42.9%) while further increments to $h = 5$ mm and $h = 6$ mm raised the peak forces to 586 N (+67.4%) and 656 N (+87.4%), respectively, with SEA at $h = 5$ mm improving to 3.3 mJm³/kg (+69.2%). The highest peak forces occurred at $h = 7-8$ mm, reaching 675-685 N (+92.9–95.7%) compared to $h = 2.8$ mm.

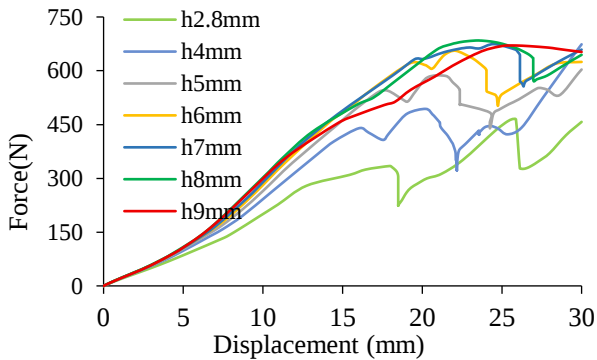


Fig. 10 Comparison of the force vs displacement curve of different initial beam heights

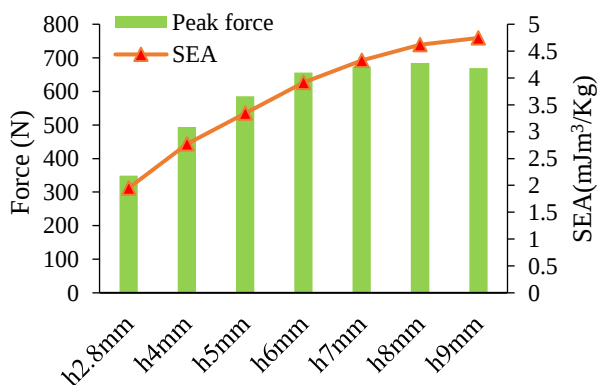


Fig. 11 Comparison of the first Peak Force and Specific Energy Absorption of differential initial beam height models

Correspondingly, SEA values also peaked in this range, with $h = 8$ mm yielding 4.62 mJm³/kg (+136.9%). At $h = 9$ mm, the peak force slightly decreased to 670 N but SEA reached 4.75 mJm³/kg (+143.6%). Overall, these results confirm that increasing beam height significantly improves load capacity and energy absorption, with $h = 8$ mm offering the most effective balance, while further increases lead to performance stabilization.

6. Discussion

The study demonstrated that geometric modifications significantly influence the flexural behavior of negative stiffness honeycomb cores. Local beam thickening improved peak load and energy absorption compared to the constant-thickness baseline. Among the modified models, LM2 provided the most balanced performance, combining enhanced load capacity with stable deformation, while LM3 achieved the highest peak force but suffered from abrupt post-peak drops, indicating brittle-like tendencies. Beam height variation further revealed that increasing height up to 7-8 mm considerably enhanced both peak load and SEA, with performance stabilizing beyond this range. These findings confirm that carefully tuned geometric parameters can delay snap-through, improve energy dissipation, and optimize structural resilience, aligning with previous studies on graded and architected honeycombs.

7. Conclusion & Future work

7.1 Conclusion

This study successfully investigated the bending behavior and energy absorption characteristics of variable-thickness negative-stiffness honeycomb cores through both experimental and numerical approaches. By systematically modifying local beam thickness and initial beam height, the research demonstrated significant improvements in mechanical performance, particularly in mitigating snap-through buckling and enhancing specific energy absorption. The key findings include:

- Finite Element Analysis (FEA) closely matched experimental results, validating the simulation model and reinforcing its reliability for future design optimization.
- Local beam thickening (LM3 configuration) provided the most balanced performance, delaying instability while maintaining high energy absorption.
- Increasing initial beam height up to 8 mm yielded optimal results, with peak force and SEA improvements exceeding 90% and 140%, respectively, compared to the baseline.

These outcomes highlight the potential of functionally graded honeycomb structures in advanced engineering applications, especially where recoverable energy absorption and structural stability are critical. The insights gained from this work pave the way for further exploration into metamaterial-based core designs, offering promising avenues for aerospace, automotive, and impact protection systems.

7.2 Future Recommendations

In this study, local beam thickness and initial beam height of double curved beam honeycomb core are varied and optimised separately for higher energy absorption and delaying snap-through buckling. For further modification, 2 measures can be taken, which are a) adding different skins to

the core to make a sandwich structure, and b) combining both functionally varied local thickness and initial beam height of the core.

8. References

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