

Impact of Piezoelectric Material Length on Piezoelectric Cantilever Energy Harvesters: A Numerical Investigation

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ABSTRACT

The utilization of piezoelectric cantilever beam energy harvesters is widespread for extracting strain-based energy from vibrations. Ongoing research focuses on enhancing the power output of these energy harvesters. Piezoelectric material length is a vital variable that warrants investigation as it affects the output power. This study investigates the effects of changing the piezoelectric material's length while keeping the base beam and piezoelectric component's thickness and width constant. While maintaining a fixed base beam length, the inquiry modifies the length of the piezoelectric material. The piezoelectric material was positioned at the fixed end of the beam to attain maximum output power for each configuration. Furthermore, due to the dependence of the output power of piezoelectric energy harvesters on the optimal electrical load and resonance frequency, this study also analyzes these contributing elements. The result indicates that the maximum power density of $12.38 \mu\text{W}/\text{mm}^3$ is achieved at a resonant frequency of 40.7 Hz when the piezoelectric material spans the entire beam length. In contrast, the lowest power density, $10.46 \mu\text{W}/\text{mm}^3$, is observed when the resonant frequency reaches 56 Hz for a piezoelectric-to-beam length ratio of 2. These findings highlight that the highest power density, which is also cost-effective, is achieved by coating either a section near the fixed end or nearly the entire surface of the beam. When piezoelectric material is applied to half of the beam, the power density significantly decreases. The resonant frequency of the energy harvester exhibits a completely opposite trend compared to the output power density as the length of the piezoelectric material changes. Decreasing the length of the piezoelectric material leads to a reduction in capacitance, thereby causing an increase in optimal resistance.

Keywords: Piezoelectricity, Cantilever beam, Energy Harvesting, Piezoelectric Material.



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1. Introduction

Energy harvesting has gained significant attention as a sustainable alternative to traditional fuel-based power generation, focusing on environmentally friendly energy sources. In this process, ambient energy—such as sunlight, variations in temperature, and mechanical vibrations—is captured and transformed into small amounts of electrical energy. Because of their higher energy conversion efficiency, piezoelectric cantilever energy harvesters have gained popularity among the different methods for collecting mechanical vibrations. These devices exploit the piezoelectric effect to convert strain energy induced by vibrations in the piezoelectric material into electrical energy. Roundy et al. [1] found that cantilever beams are especially effective for vibration energy harvesting, as they experience higher average strain compared to other structural configurations.

The geometry of the energy harvesting device significantly influences the output power density. Researchers continue to focus on improving both the output power density and bandwidth, as maximum power is typically achieved near the resonant frequency. Zhang et al. [2] explored beam geometries and found that trapezoidal beams outperform rectangular ones. Various cantilever beam shapes, such as rectangular, trapezoidal, and inverted trapezoidal, were investigated in a study by Pradeesh and Udhayakumar [3]. According to their research, the highest output power is produced by a tapered inverted trapezoidal beam. Another study by Mohiuddin et al. [4] also suggested an inverted

trapezoidal beam, especially when the beam is coated partially. They also found that this configuration is the most cost-effective [5]. Their findings suggested that a tapered inverted trapezoidal beam provides the highest output power. Alameh et al. [6] introduced an innovative T-shaped design for piezoelectric energy harvesters to improve performance. In another study by Mohiuddin et al. [7] demonstrated that convex cantilever shape outperforms conventional rectangular beams. By investigating unimorph piezoelectric cantilever Sunithamani et al. [8] discovered that a disc-shaped proof mass performed more efficiently than a ring-shaped one. Additionally, Wang et al. [9] investigated the effectiveness of unimorph cantilever energy harvesters and how it relate to the thickness ratios of the beam and piezoelectric layers. Erturk et al. [10] showed that using segmented electrodes, instead of continuous ones that cover strain nodes, significantly improves voltage output by avoiding the cancellation of electrical output at strain nodes.

Pradeesh and Udhayakumar [11] also examined how the form of the proof mass and the placement of the piezoelectric layer along the cantilever beam affected output power. According to their findings, the position of the piezoelectric material at the fixed end of the beam produces the maximum output power for a particular form. Tang and Wang [12] focused on how the size of the proof mass affects the performance of energy harvesters. They came to the conclusion that resonance frequency, strain distribution, and output power can all be greatly impacted by even little

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modifications to the proof mass geometry. Zhou et al. [13] studied the effect of varying the length of the piezoelectric material along the beam, showing that optimizing its length enhances energy harvesting performance. Izadgoshasb et al. [14] demonstrated that human motion vibrations could be utilized to improve the efficiency of the piezoelectric cantilever energy harvester. Benasciutti et al. [15] designed a cantilever energy harvester for wireless sensor networks, finding that a trapezoidal geometry with clamping at the wider side was the most effective.

While considerable efforts have been made to optimize the geometry of cantilever energy harvesters, there is still a need for a detailed analysis of the length of piezoelectric material. Since the cost of piezoelectric material is significantly higher than that of the beam, optimizing its length as a ratio of the beam length is crucial. This study investigates how changes in the length of piezoelectric material affect the harvester's performance, with the beam length held constant. The output is measured in terms of power density relative to the material's volume, enabling a fair comparison across different piezoelectric material lengths, as material cost correlates with volume. Since maximum power production occurs at resonance frequency and optimal load, the paper also investigates how the variation of piezoelectric material length causes the variation of resonant frequency and optimal electrical load.

2. Fundamental Equations

The following governing equations explain the relationship between strain and the electric field in a piezoelectric material..

$$\sigma = C_E \varepsilon - e^T E \quad (1)$$

$$D = e \varepsilon + \varepsilon_0 \varepsilon_{rs} E \quad (2)$$

C_E , e , and ε_{rs} are the properties of material. To solve the piezoelectric problem, the equations from solid mechanics and electrostatics, in addition to Equation (1) and Equation (2), must be considered.

Constitutive relationship of electric displacement

$$D = \varepsilon_0 E + P \quad (3)$$

Here, ε_0 represents the vacuum permittivity, and P denotes the electric polarization vector.

The charge density relationship can now be stated as follows.

$$\rho_v = \nabla \cdot D \quad (4)$$

The equation of equilibrium in solid mechanics is given by,

$$\rho \frac{\partial^2 u}{\partial t^2} = \nabla_x P^T + F_V e^{i\varphi} \quad (5)$$

Here, F_V represents the body force components, and P denotes the first Piola-Kirchhoff stress tensor.

3. Configuration and Constraints

COMSOL Multiphysics was used for the study, incorporating modules for electrical circuits, solid mechanics, and electrostatics. The solid mechanics module resolved the equations of motion to evaluate stress and strain under different loading conditions. The electrostatics module computed the electric and displacement fields, along with potential distributions within piezoelectric materials. Additionally, the electric circuit module was employed to simulate current and voltage responses in the circuit.

The setup included a cantilever beam that was free at one end and fixed at the other, as shown in Fig. 1. The beam, constructed from aluminum, featured a 0.5 mm thick layer of piezoelectric material (PZT 5A) applied at the fixed end in a unimorph arrangement. This configuration was chosen based on its ability to maximize power output, as recommended by Pradeesh and Udhayakumar [8]. The piezoelectric coating and the aluminum base were both consistently 10 mm wide. At the free end, a 0.17 g proof mass was attached and damping losses were isentropic with a 5% value. [11]. To account for the load-dependent nature of piezoelectric energy harvesters, each beam was analyzed using its optimal electric load. To find each beam's mode-1 resonant frequency, the study first calculated the eigenfrequency. A frequency domain analysis was then conducted, exposing the beams to varying input frequencies to examine changes in power density near resonance. Power density, or output power (μW) divided by the volume (mm^3) of the piezoelectric layer, was used to quantify the energy harvester's performance.

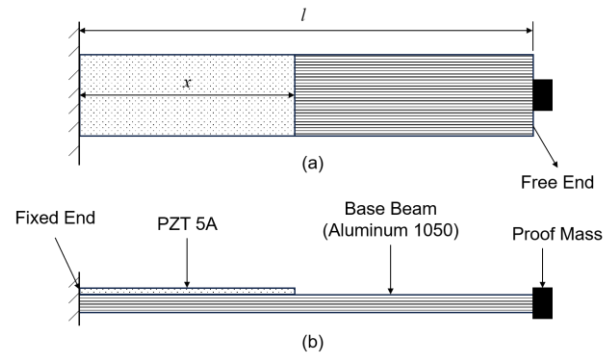


Fig.1 Piezoelectric Cantilever energy harvester configuration: (a) top view, (b) side view

The energy harvester's configuration is shown in Fig.1, where 'l' and 'x' stand for the base beam and the piezoelectric material's length, respectively. The entire harvester was subjected to a gravitational force of 1g, which was systematically varied in a harmonic pattern.

4. Model Validation

A rectangular cantilever beam piezoelectric energy harvester measuring 100 mm in length, 10 mm in width, and 1 mm in thickness was utilized to verify the simulation model. On the fixed end of the beam, a piezoelectric layer with dimensions of 10 × 10 × 0.5 mm in length, width, and 0.5 thickness was applied. The frequency domain analysis covered a range of 70 Hz to 110 Hz to observe how output power changed over this spectrum. A maximum output of 0.37 mW at 94.3 Hz was obtained. Comparing these findings with those reported by Pradeesh and Udhayakuma [8] in the literature, the results are consistent, thereby validating the simulation methodology used in this study.

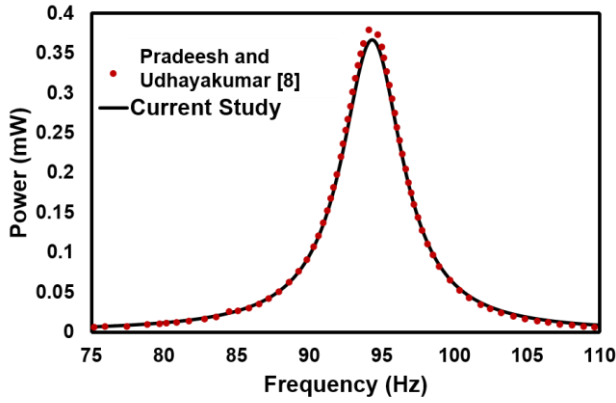


Fig.2 Frequency-dependent variation of output power (in mW) for computational procedure validation.

5. Results and Discussion

To analyze how the output power density is affected by the length of the piezoelectric material, the base beam's length was kept constant at (l') 50 mm. Concurrently, the ' l/x ' ratio was employed to change the length of the piezoelectric material, which varied in steps of 0.25 from 1 to 3. Consequently, the investigation commenced with the same base beam and piezoelectric material's length of 150 mm. As the ratio reached 3, the piezoelectric material's length gradually decreased to 50 mm. The relationship between the output power density and the length of the piezoelectric material is shown in Fig.3. When the ratio is 1, the maximum power density of $12.38 \mu\text{W}/\text{mm}^3$ at 40.7 Hz is achieved. Initially, the power density decreases as the ratio increases up to 2, reaching its lowest point at $10.46 \mu\text{W}/\text{mm}^3$ at 56 Hz. Afterward, it starts to rise again, reaching $11.4 \mu\text{W}/\text{mm}^3$ at a ratio of 3, occurring at 50.4 Hz.

Power density from the piezoelectric material is primarily dependent on the average induced stress and strain within the material. Higher induced stress correlates with higher power density, and vice versa. Understanding the stress distribution in a cantilever beam reveals that induced stress is typically higher near the fixed end, gradually decreasing towards the free end and reaching its lowest at the free end. Therefore, the part of the piezoelectric material close to the fixed end has the greatest influence on the output power density. Increasing the ratio from 1 introduces two factors affecting power density. The first factor involves reducing the mass of the system, which in turn reduces the bending moment at the fixed end of the beam, thereby reducing stress in the dominant portion of the piezoelectric material. This factor tends to reduce the power density. The second factor involves eliminating a portion of the piezoelectric material from the free end of the beam, which generates only a small amount of power. Since power density is calculated as the output power divided by the volume of the piezoelectric material, eliminating a comparatively large volume of piezoelectric material from the free end for a marginal reduction in output power may result in both increase or decrease in output power density. Initially, raising the ratio means removing piezoelectric material that is located far from the fixed end. Removing mass from a substantial distance from the fixed end reduces considerable strain in the dominant section of the piezoelectric material because the bending moment has a proportional relation to the product of mass and distance. This explains why, initially, the output power density decreases with an increase in the ratio.

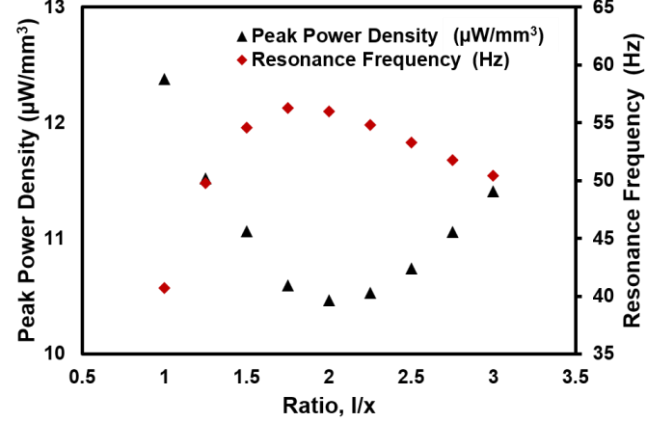


Fig.3 Fluctuations in maximum output power density and resonant frequency in relation to the ' l/x ' ratio.

However, raising the ratio beyond 2 leads to an increase in output power density. This shift occurs because, at this point, the piezoelectric material is being eliminated from a relatively shorter distance. Since the bending moment varies in proportion to the product of mass and distance, the drop in the bending moment in the dominant region of the piezoelectric material is therefore small even though the mass reduces. This is because the material is closer to the fixed end. Eliminating a substantial volume of piezoelectric material at this stage results in a relatively smaller drop in output power due to a comparatively smaller impact on stress at the dominant fixed-end portion of the beam. Consequently, power density increases as reduction in power is marginal in comparison to the eliminated volume of piezoelectric material.

This finding is crucial for developing energy harvesters with cantilever beams. Piezoelectric cantilever harvester costs depend largely on piezoelectric material volume, expensive than the base beam material. Optimal output suggests fully coating the base beam with piezoelectric material or only a small section near the fixed end. This configuration maximizes power per unit cost. Avoiding a ' l/x ' ratio near 2 is advisable to prevent a notable decrease in power density.

Additionally, Fig.3 shows an inverse relationship between resonant frequency and output power density with respect to the ' l/x ' ratio. The stiffness of the beam affects the resonance frequency in cantilever beams. The beam's resonance frequency rises with increasing stiffness and vice versa. The stiffness of the piezoelectric material is inversely related to the static deflection at the free end of the beam. However, because the piezoelectric material is shorter when the ' l/x ' ratio is increased, the energy harvester's total weight decreases. This reduction in weight can either increase or decrease the beam's stiffness, depending on which portion of the piezoelectric material is reduced or removed. Considering that the bending moment at the fixed end is proportional to both mass and distance from the free end, a more significant drop in bending moment occurs at the fixed end when the piezoelectric material is eliminated near the free end compared to near the fixed end. It is important to note that the cantilever beam's deflection curve shows higher curvature near the fixed end, gradually decreasing towards the free end. Consequently, the deflection of the free end primarily depends on the bending moment and area moment of inertia near the fixed end. Conversely, beam deflection is inversely proportional to the area moment of inertia. When piezoelectric material is eliminated from a portion of the

beam, that section tends to exhibit higher deflection curvature due to the cubic relationship of the moment of inertia to thickness. However, if piezoelectric material is removed near the free end, the deflection of the free end decreases since the bending moment near the fixed end is reduced, while the area moment of inertia remains constant.

At lower ratios, an increase in the ratio suggests the elimination of piezoelectric material near the free end, leading to a decrease in free-end deflection and an increase in stiffness and resonant frequency. This trend continues up to a ratio of 1.75. Beyond this point, a further increase in the ratio indicates that piezoelectric material is being eliminated near the fixed end. Consequently, the impact on the decrease in bending moment is relatively lower, but the section near the fixed end experiences more deflection curvature due to a decreased area moment of inertia. In this case, the effect of the area moment of inertia becomes more significant than the bending moment, resulting in a decrease in resonant frequency with a ratio increase beyond 1.75.

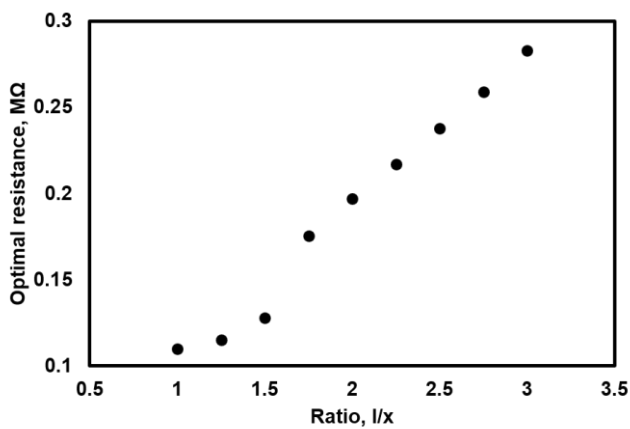


Fig.4 Variation of optimal resistance with ' l/x ' ratios.

Maximum output power happens when the applied electric load equals the internal impedance of the piezoelectric material that coats the cantilever beam, in accordance with the maximum power transfer concept. The operating frequency and the capacitance of the beam have an inverse relationship and affect this internal impedance. An increasing ratio means the capacitance reduces as the length of the piezoelectric material decreases. As the capacitance decreases with increasing ratios, Fig.4 shows that the optimal resistance for the harvester increases. While the resonant frequency shows a non-linear trend with the ' l/x ' ratio—initially increasing before declining—Fig.4 highlights that in this context, the internal impedance is primarily determined by the material's capacitance.

6. Conclusion

This study examines how changing the piezoelectric material's length affects the piezoelectric cantilever energy harvesters' output power density while maintaining a constant base beam length. The results indicate that although the output power decreases linearly with a reduction in piezoelectric material length, the variation in output power density is non-linear. Output power density is higher at both lower and higher ratios (corresponding to shorter and longer lengths of piezoelectric material), with a notable drop for ratios near 2. Specifically, the maximum power of $12.38 \mu\text{W}/\text{mm}^3$ occurs at a natural frequency of 40.7 Hz when the piezoelectric material length equals the beam length, while

the minimum power of $10.46 \mu\text{W}/\text{mm}^3$ is observed at a natural frequency of 56 Hz for a piezoelectric-to-beam length ratio of 2. For a short piezoelectric material length corresponding to a ratio of 3, the power density recovers to $11.4 \mu\text{W}/\text{mm}^3$ at a resonant frequency of 50.4 Hz, demonstrating the potential for cost-effective designs that retain reasonable performance. The results highlight that longer piezoelectric material is preferable for low-frequency environments, while shorter lengths near the fixed end are better for higher-frequency applications and cost-sensitive designs. Given the significantly higher cost of piezoelectric material compared to the host beam material, the results suggest it is more cost-effective to apply the piezoelectric material over most of the beam when cost is not a major concern or to focus on placing a shorter coating near the fixed end when prioritizing cost efficiency. The change in resonant frequency can be compensated by modifying the proof mass. Furthermore, the removal of piezoelectric material with an increase in ratio initially results in an increase in resonance for lower ratios due to a reduction in bending moment, followed by a decrease in resonance for higher ratios due to a decrease in the area moment of inertia. The elimination of piezoelectric material with an increase in ratio leads to a reduction in capacitance, resulting in an increase in optimal resistance. While this study provides insights into optimizing piezoelectric material length for energy harvesters, future work could explore the impact of piezoelectric material length on the durability and fatigue performance of the cantilever over time, particularly in applications requiring long-term reliability.

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