An Efficient Computational Technique for the Analysis of Telegraph Equation

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ABSTRACT

The Telegraph equation has drawn much attention due to its recent variety of applications in different areas of the communication system. Various methods have been developed to solve the Telegraph equation so far. In this research paper, we have formulated a derivation mathematically for the Telegraph equation for the section of a line of transmission concerning the voltage associated and the current. Therefore, obtained mathematical equation has been solved numerically by COMSOL Multiphysics. We have then numerically analyzed the parametric behavior of the Telegraph equation. The analysis first starts with allowing both the damping coefficients to vary, keeping the transmission velocity fixed, and observing the pulse shape at different time slots. We have then investigated the deformation of the pulse caused due to the gradual increase of transmission velocity for varying damping coefficients at the intended discrete time slots. Finally, we analyzed the behavior of the associated voltage pattern for those variations due to the corresponding distance of the Telegraph wire. We have observed that changes in the damping coefficients have a gradual impact on the associated voltage of the Telegraph equation, which is more conspicuous for the higher time slots. Transmission velocity is found as the most influential parameter of the Telegraph equation that controls the deformation of the pulse height, which is the cardinal part of the inquiry.

Keywords: Damping Coefficients, Transmission Velocity, Time Propagation, Voltage Drop, Pulse Height, COMSOL Multiphysics, Numerical Simulation.

1   Introduction

In this modern age, we need a high-frequency communication system and this system needs transmission media for transferring a signal from one point to another. We can categorize the transmission media into two groups, viz. guided and unguided media. Signals through the guided medium are transferred through the transmission line or the coaxial cable. But in the case of unguided media, the signals partly or entirely through the path of communication are carried by electromagnetic waves and are carried through the radio frequencies (RF) and microwaves (MW) communication channels. Transmission and reception of electromagnetic waves are done through the antenna. For addressing the problem of efficiency of telegraphic transmission in the case of guided medium, we investigate the cable transmission medium specifically. Since all the systems incur losses, optimizing the system of communication with guided, therefore the determination or power of the project along with the losses of signals is essential. For the determination of loss and finally for ensuring the output with maximum value, we need the determination of the losses, and these losses are to be calculated with some equations which are to be formulated essentially.

1.1   Historical Background of Telegrapher's Equations

The Telegraph equations are comprised of differential equations in linear pairs. These equations express the voltage and current through a line of transmission with electricity based on the distance covered and the time spent. These equations had been followed since Oliver Heaviside developed the model of the line of transmission in the 1880s, and

that is what is described in this paper [1]. The objective model is to demonstrate the waves related to electromagnetic that can have reflection through the wire. Also, these patterns of waves may have their appearance along these lines [2]. This concept may apply to the lines of transmission regardless of frequency that is for lines of transmission with frequency in high (for example, wires of the telegraph, conductors corresponding to frequency of radio), frequency of audio (e.g., lines of telephone), and frequency is low (e.g., lines of power). And it is also applicable for direct current (DC) [3].

S. A. Yousefi applied a numerical method based upon Legendre multiwavelet approximations for solving the one-dimensional hyperbolic telegraph equation. He utilized the properties of the Legendre multiwavelet along with the Galerkin method to reduce the telegraph equation to the solution of algebraic equations [4]. M. Lakestani and B. N. Saray solved the Telegraph equation using interpolating scaling functions and reduced the equations to a set of algebraic equations using the operational matrix of derivatives [5]. M. Dehghan and A. Ghesmati developed a new method based on the unification of fictitious time integration (FTI) and group preserving (GP) methods and applied it to solve the Telegraph equation [6]. V. K. Srivastava *et al.* deduced the exact solution or a close to the exact solution of the differential equations by the reduced differential transform method and applied it to the Telegraph equation too (RDTM) [7]. R.C. Mittal and R. Bhatia numerically solved the one-dimensional hyperbolic telegraph equation by the B-spline collocation method. The method is based on the collocation of modified cubic B-spline basis functions over finite elements [8]. R. C. Tautz and I. Lerche introduced a closed-form analytical technique for solving the three-dimensional Telegraph equation

and implemented it for cosmic-ray transport [9]. G. Arora and V. Joshi obtained the approximate solution of the telegraph equation with two different modified spline basis functions by the differential quadrature method and computed the weighting coefficients to transform the equation into a set of first-order conventional differential equations that were again solved by the SSP-RK43 method and compared the convenience of the methods applied [10]. S. N. Hussen numerically solved the one-dimensional hyperbolic telegraph equation by using the q-homotopy analysis method (q-HAM) and obtained greater convergence compared to the homotopy analysis method (HAM) [11]. Z. Stojanović and E. Čajić observed the phenomena occurring in one part of the electromagnetic spectrum in the frequency range of 1 GHz-100 GHz as an aggravating factor in signal transmission by telecommunication water [12]. H. Khan et al. implemented a decomposition method-based analytic approach utilizing the natural transformation. Caputo operator is incorporated for the fractional derivative and applied for the analytical treatment of the solution of fractional-order hyperbolic Telegraph equation. Some practical examples are assigned for the justification of their work [13]. M. S. Hashemi applied the technique of Shape functions extracted from the moving Kriging (MK) interpolation to the weak form of the telegraph equation (TE) in space coordinates and got a system of second-order ordinary differential equations concerning the time variable. The resultant system was solved by the group preserving scheme (GPS) [14]. Y. Zhou et al. derived a hybrid meshless method for solving the second-order telegraph equation in two-space dimensions incorporating the Dirichlet or mixed boundary conditions. They successfully applied and validated their work for a meshless finite difference method for solving and analyzing the second-order Telegraph equation [15].

In this work, we have analyzed the behavior of the voltage pattern of the Telegraph wire due to the variations of the corresponding parameters, namely, damping coefficients and transmission velocity for several discrete time slots. The solution of the Telegraph equation is not aimed but rather the impact of the parametric variations. The investigation is incorporated numerical simulation, graphical representation, and narrative analysis. A case-by-case discussion is conferred at the end of the work.

1.2 Distributed Components

The Telegraph equation, like other equations that describe phenomena in electrical, is resulted from Maxwell's equations [16]. In the practical application of engineering fields, it is assumed about the conductors that they comprise components with two-port and a series of infinite elementarily and each component represents a segment that is short and very infinitesimal to the line of transmission [17]. For the convenience of the description of the components and attributes of the Telegraph equation, Table 1 is indispensable, which includes the interpretations of various symbols and notations that are associated with this work.
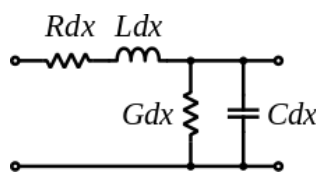

Fig. 1 Schematic depiction of the distributed components of a line of transmission [18].

Fig. 2 Schematic depiction of the wave flowing through a transmission line [18].

In Fig. 1, the distributed resistance R for conductors has been represented with series of resistor which expresses length per unit in terms of ohms. The inductance of distribution L for the field of magnetic around the wire, self-inductance, etc.) has been represented with series of inductor (Henries of length for per unit). The capacitance C has been illustrated with a capacitor (farads of length per unit) of the shunt that is between conductors and two in numbers. The conductance G of dielectric materials that separate conductors has been represented with a resistor of a shunt between a wire of signal and a wire of return (siemens of length per unit). Resistor associated with the resistance $\frac{1}{G}$ ohms [19]-[21].

Table 1 Nomenclature of the symbols and notations.

Symbol/Notation	Name of the component/parameter
R	Distributed resistance for the conductors
L	The inductance of the distribution
C	The capacitance of the dielectric materials
G	Conductance of the dielectric materials
v	Associated voltage in the cable
i	Associated current in the cable
x	Distance corresponding to the end from where the cable sends
t	Time for current flow
p	Partial differential equation coefficients or the damping coefficients
q	
r	Transmission velocity
dx	Increment in distance
v_x	The first-order partial derivative of v concerning x
v_t	The first-order partial derivative of v concerning t
v_{xx}	The second-order partial derivative of v concerning x
v_{tt}	The second-order partial derivative of v concerning t

Fig. 2 depicts the wave flowing from the rightward direction to downward through a lossless line of transmission where the black dot represents electrons. The field of electricity is shown by drawing arrows. The components with their role based on the animation can be visualized on the right side.

The model that we have proposed is consisting of components of elements that are infinitesimal and with a series of infinite series of the infinite are depicted in Fig. 1. Also, we have specified their values by the length of the unit. Also, we treat the mentioned quantities as primary line constants so that they can be distinguished from secondary line constants, all of them are impedance of characteristic, constant of propagation, constant of attenuation, and constant of phase. They are the constants concerning current, voltage, and time. These can be treated as the frequency of functions that are not constants.

1.3 Role of Different Components

The inductance L happens interestingly that the electrons seem to have inertia, which means that the flow of current cannot be increased or decreased easily with a large inductance for any points given. Inductance with a large amount causes waves to move even much slowly, same as that wave can travel much slowly downward a rope with a heavy size than that of one with light size which can give it larger impedance (relatively current with a lower amount for identical voltage). Capacitance C has control over the bunched-up electrons of bunched-up on the point that how much they repel each other, and also on the point of attraction of the spread-out electrons that how much they attract each other conversely. Attraction and repulsion for the capacitance of large value become less due to another line that usually contains the charge of opposite character and balances partly the force of attraction or repulsion. Alternatively, it can be said that the identical charge build-up causes less amount of voltage for the larger capacitance (i.e., the force of weak restoring), which causes waves to move slowly and give the impedance of a lower amount (i.e., the voltage of lower amount for the current of the same value). R is corresponding to the resistances within lines whereas G allows the current to flow to and from one line to another. Fig. 1 shows the lossless line of transmission, whereas both R and G taken to be zero.

2 Numerical Investigations of the Attributes of Telegraph Equation

In this section, numerical analysis of the behavior of the Telegraph equation is observed. Model setup along with the initialization of the parameters and the boundary conditions with the initial condition for the desired Telegraph equation is discussed.

2.1 Model Setup

We have assumed a wire piece with cable of the Telegraph has infinitesimal and also assumed it to be a circuit of electricity which is demonstrated in Fig. 3. Furthermore, we assumed this cable to be insulated imperfectly for which the existence of capacitance along with the leakage of current to the ground is observed [22].
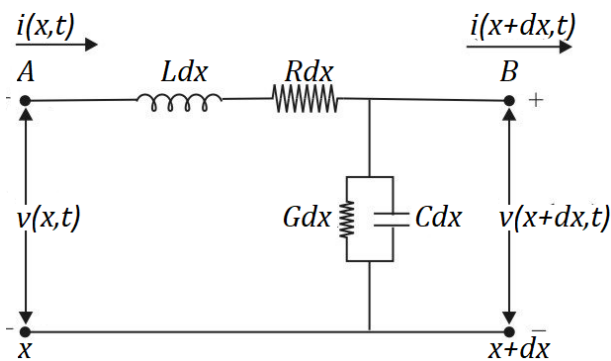

Fig. 3 Schematic depiction of telegraphic transmission line involving leakage [23],[24].

In Fig. 3 assume that x be the distance corresponding to the end from where the cable sends; $v(x,t)$ be the associated voltage with time t for points; $i(x,t)$ be the associated current on the cable with time for points, also R be the cable's resistance; C be the capacitance that is associated to the ground; L be the cable's inductance, and G be the conductance that is associated to ground.

Therefore, the voltage using the laws of Ohm across the resistor can be written as

$$v = iR. \tag{1}$$

Now, the drop of voltage across the inductor is the following

$$v = L\frac{di}{dt}. \tag{2}$$

Across the capacitor, the drop of voltage is given by

$$v = \frac{1}{C}\int i dt. \tag{3}$$

Now, the voltage at the terminal B equals the voltage of the terminal at A, subtraction the voltage drop along considered element AB, therefore combining Eqs. (1), (2), and (3) together, it can be written as

$$v(x + dx, t) - v(x, t) = -[Rdx]i - [Ldx]\frac{\partial i}{\partial t}. \tag{4}$$

Let $dx \to 0$ then partial differentiation of Eq. (4) concerning x yields

$$\frac{\partial v}{\partial x} = -Ri - L\frac{\partial i}{\partial t}. \tag{5}$$

Likewise, the current at the terminal B equals to current at the terminal A subtraction current throughout the leakage to the ground, then it is found that

$$i(x + dx, t) = i(x, t) - [Gdx]u - i_c dx. \tag{6}$$

Now, the current throughout the capacitor can be written as

$$i_c = C\frac{\partial u}{\partial t}. \tag{7}$$

Now, differentiating Eqs. (4) and (7) concerning x and t, respectively. We eliminate the derivatives of v. Finally, we get the followings

$$r^2 \frac{\partial^2 i}{\partial x^2} = \frac{\partial^2 i}{\partial t^2} + (p + q)\frac{\partial i}{\partial t} + (pq)i, \tag{8}$$

$$r^2 \frac{\partial^2 v}{\partial x^2} = \frac{\partial^2 v}{\partial t^2} + (p + q)\frac{\partial v}{\partial t} + (pq)v, \tag{9}$$

where $p = \frac{G}{C}$ and $q = \frac{R}{L}$ are positive constants and are known as damping coefficients, $r = \frac{1}{\sqrt{LC}}$.

Equations (8) and (9) comprise the aimed one-dimensional hyperbolic second-order Telegraph equation [25].

2.2 Parameter Setup

We can use this model of the Telegraph equation to investigate how pulses of voltage are transmitted through the Telegraph wires. This model of the Telegraph equation can model mixtures between the diffusion and the propagation of a wave that can introduce the term which causes the standard heat or equation of transport of mass for effects of finite velocity [26]. This example is modeled for a Telegraph wire of a small section and it involves the study of the pulse's voltage while it moves along it. The results that cause the varying damping coefficients are sketched with the shape of the pulse and are provided by a parametric sweep.

It is very simple to define the model where the geometry consists of a line of length 1 which is one-dimensional. The

initial condition is taken as a voltage distribution that is bell-shaped and thus the pulse is modeled. The conditions corresponding to the boundary define flux at both ends for the section of the wire and these conditions allow freely varying the voltage.

The Telegraph equation (Eq. (9)) can be written in a more compact form as

$$v_{tt} + (p + q)v_t + pqv = r^2 v_{xx}. \tag{10}$$

Here, v, the dependent variable denotes the voltage, whereas x, the independent variable denotes the distance from the initial position of the Telegraph wire. The model has been initiated for some fixed parameters that are for $p = 0.5 = q$ and $r = 1$.

2.3 Boundary Conditions and Initial Condition

At both ends, the boundary conditions are Neumann conditions and are also homogeneous as

$$v_x(t, 0) = 0, \quad v_x(t, 1) = 0. \tag{11}$$

The initial condition consists of the following equations which are describing a bell-shaped pulse and has the highest point at 0.2 and also a base width of 0.4.

$$v(0, x) = e^{-3\left(\frac{x}{0.2} - 1\right)^2}, \quad v_t(0, x) = 0. \tag{12}$$

3 Results and Discussion

This section investigates the important mathematical properties of the Telegraph equation. The following is aimed to provide insight into the behavior of the Telegraph equation. In particular, we analyze the characteristic structure of the Telegraph equation. We will interpret graphically in one-dimensional space. Here, we applied COMSOL Multiphysics (version 4.3) for the numerical computation in a Windows machine having an Intel i5-6200U CPU with 2.30 GHz clock speed, 2 cores each, and 8 GB of total RAM.

We have studied the effect of the variations of the partial differential equation coefficients (damping coefficients) p and q, and the transmission velocity r on the pulse's shape (length vs voltage graph) for several discrete time slots. In Eq. (10), it is observed that the co-efficient of voltage and its rate of change about the time both are symmetric. They exhibit the variations attributes for the changes of the $p + q$ and pq, not the individual changes of p and/or q. But there are significant variations found for the changes in transmission velocity r over time specified time slots.

In this section, we are investigating the effect of the changes in the transmission velocity r for some discrete time slots with some particularized values of damping coefficients p and q. We are exploring the deformation of the pulse shape of the Telegraph equation due to the desired variations.

In the following case studies, we are scrutinizing the variation of voltage (v) for the damping coefficients p and q, ranging $p + q = 1, 2, 3, 4$ for the transmission velocity $r = 1, 2, 3$ and discrete time slots ranging $t = 0.25, 0.5, 0.75, 1$. These observations are attained from numerical simulations and analyzed graphically.
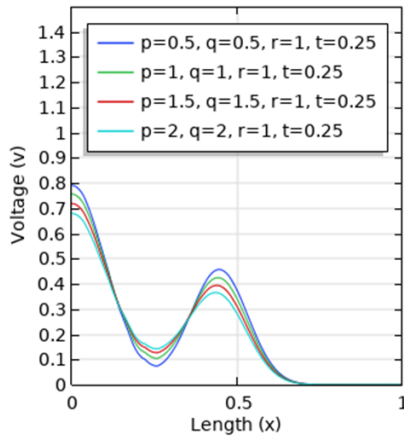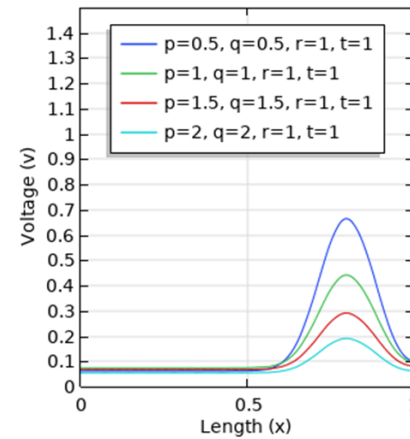

(a) Transmission velocity $r = 1$ and time $t = 0.25$
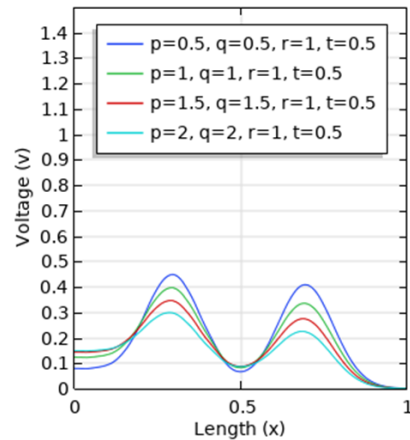
(b) Transmission velocity $r = 1$ and time $t = 0.5$
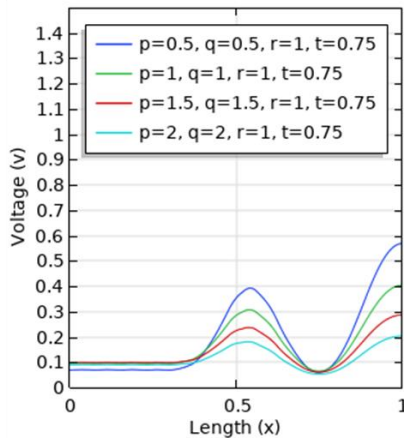
(c) Transmission velocity $r = 1$ and time $t = 0.75$

(d) Transmission velocity $r = 1$ and time $t = 1$

Fig. 4 Different pulse shape for $r = 1$ and $t = 0.25, 0.5, 0.75, 1$.

3.1 **Case-1:** Transmission Velocity $r = 1$ and Time Slot $t = 0.25, 0.5, 0.75, 1$

We have investigated the voltage variation for the damping factors i.e. $p + q$, ranging from 1 to 4 for the transmission velocity $r = 1$, and time slots ranging from $t = 0.25$ to $t = 1$. Our curiosity was to observe how the changes are increasingly occurring at several times for the same value of r and intentionally allowed the damping factor to vary. The consequences observed are numerically sketched in Fig. 4.

In Fig. 4(a), we have depicted the pulse shape for a specific time slot $t = 0.25$, $r = 1$, the damping coefficients starting from $p = 0.5 = q$ to $p = 2 = q$ with evenly spaced and enhanced. A significant variation of decrease of pulse height was observed at nearly quarter length of the propagation of the transmission. The increase in damping coefficients caused the pulse to increase again for a small length of the transmission line and then with a dramatic decrease in the height of the pulse height.

The increase of time for the same transmission velocity and the gradual variation of p and q causes an interesting observation as shown in Fig. 4(b). For the time slot $t = 0.5$, we observed a symmetric up and down of the pulse with the symmetric variation in between the damping factors themselves. A smooth variation with the fashion of increase and decrease of pulse with the propagation of the transmission makes a sensation in the observation. Exactly at the middle of the propagation of the length of the wire, there is a significant variation of the down of the pulse, and an evenly spaced increased pulse was observed.

The pulse has applied the same ratio of height for the varying damping factors. A smooth sequence of observations enhanced the curiosity to increase the time with the fashion of varying damping coefficients and with the fixed transmission velocity.

With the enhancement of time, we observed that the deformations are quickly occurring in the pulse shape. Transmission velocity $r = 1$ and time $t = 0.75$ with the gradual enhancement of damping coefficients in an evenly spaced manner causes deformation pronounced significantly at the initial point of the graph. It seemed that there is a coincidence with more than two pulse heights coming from different damping parameters and as the transmission propagates along the wire, the pulse height starts increasing at nearly the middle of the length. The height then went down again and seemed to intersect at a certain point. And the interesting matter happens later when the height of the pulse increases to a greater extent with the varying parameters of the values of p and q. These variations and insight attributes have been analyzed in Fig. 4(c).

While quite eye-catching variations were noticed in Fig. 4(d) for the time slot $t = 1$. Here, the deformation is observed at a quite distance length of the transmission. We interestingly observed the very nearest coincidence of the propagation of the transmission of all the damping factors although were varying at a value of $p = 0.5 = q$ for the fixed time slot and fixed transmission velocity up to a very wide length of the wire. The pulse increased later maintaining a ratio among the factors occurring for different curves and then damped down again.
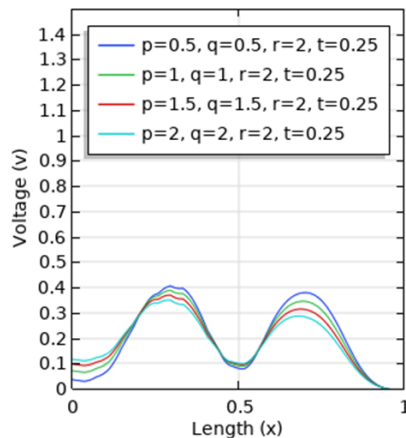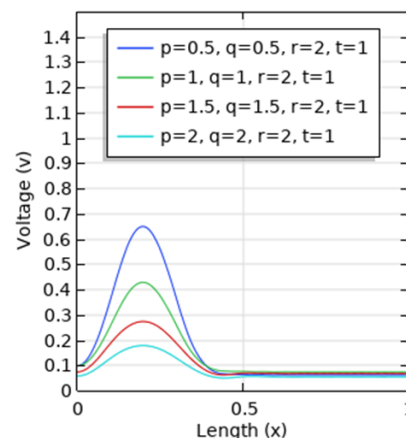

(a) Transmission velocity $r = 2$ and time $t = 0.25$
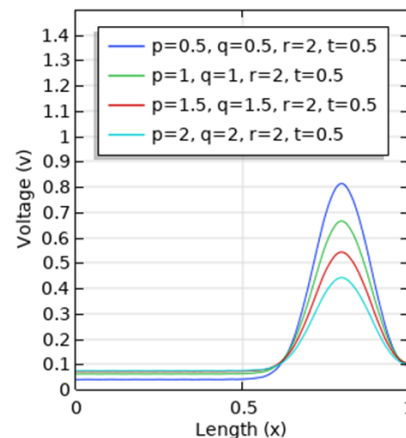
(b) Transmission velocity $r = 2$ and time $t = 0.5$
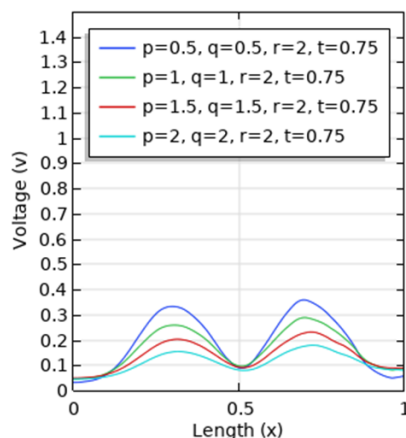
(c) Transmission velocity $r = 2$ and time $t = 0.75$

(d) Transmission velocity $r = 2$ and time $t = 1$

Fig. 5 Different pulse shape for $r = 2$ and $t = 0.25, 0.5, 0.75, 1$.

3.2 Case-2: Transmission Velocity $r = 2$ and Time Slot $t = 0.25, 0.5, 0.75, 1$

Our first observation for the time slot $t = 0.25$ for the earlier stated case-2 parameters variation is found in Fig. 5(a). A massive variation was observed at the beginning. Pulses emerging at a specific ratio with increasing pattern cause damping up symmetric behave and similar damping down symmetric exactly at the middle length propagation of the wire. It produced a similar increasing smooth pulse later with the propagation of time. A massive sensation of the pulse was observed in this case. A variation quite different from Fig. 5(a) is observed in Fig. 5(b) due to the increase of time and specified time slot $t = 0.5$. A constant pulse height happened more than halfway through propagation. A sudden and significant pulse height was observed for the damping varying factors later and damping down height smoothly. At time slot $t = 0.75$, a comprehensive observation followed in Fig. 5(c). There observed a symmetry shape of pulse height in the neighborhood of the middle length of propagation and the converging of the height at a point when the transmission propagates the half-wire length. The smoothness of the curve was also observed more soundly. Fig. 5(d) depicts the pulse shape for case-2 for the time slot $t = 1$. We observed a scattered shape at the beginning and a coincidence emerging from the half-length propagation.

3.3 Case-3: Transmission Velocity $r = 3$ and Time Slot $t = 0.25, 0.50, 0.75, 1.00$

Likewise, in case-1 and case-2, for the fixed time slots and variation of damping coefficients with a high velocity of transmission $r = 3$ was numerically observed and is described in Fig. 6. The increase of transmission velocity and a gradual increase of damping factors occurring with the desired time slots caused the scattered pulse height as shown in Fig. 6. It is kept mild for $t = 0.25$ and $t = 0.5$, and then a more scattering pulse heigh is found for $t = 0.75$. The highly scattering pulse height is seen in Fig. 6(d) at the propagation for the time slot $t = 1$. An interesting matter happened in the time slot $t = 0.5$ as shown in Fig. 6(b), and a scatter pulse with a smooth and symmetric deformation is observed.

We curiously continued the simulation for the increasing time and observed the pulse height was scattering significantly with the significant increase of the damping factors along with the desired time slots for high transmission velocity. The scattering phenomenon is widened with time.
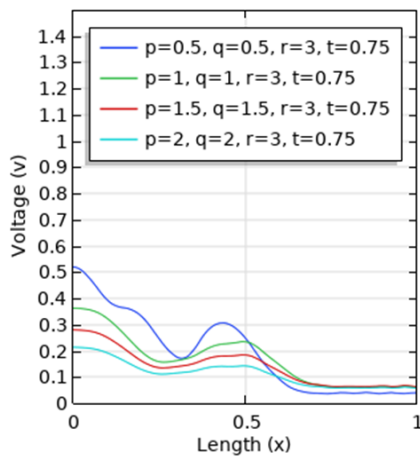
Now, it is evident that the transmission velocity plays a significant role in the analysis of variation of damping coefficients over the pulse height for the propagation along the Telegraph wire.
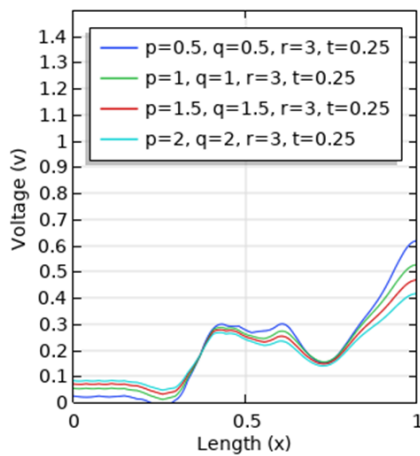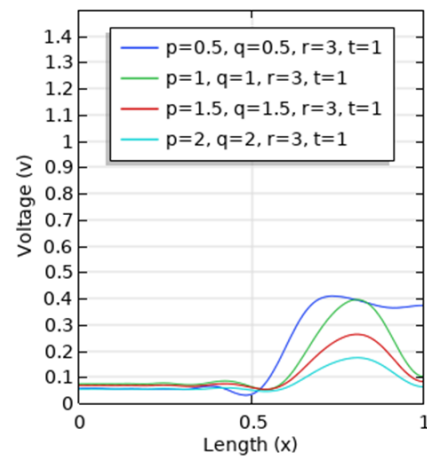

(a) Transmission velocity $r = 3$ and time $t = 0.25$
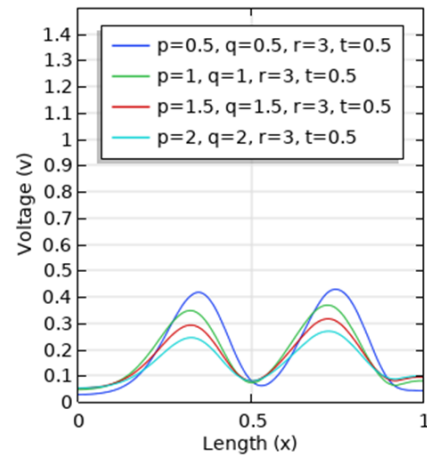
(b) Transmission velocity $r = 3$ and time $t = 0.5$

(c) Transmission velocity $r = 3$ and time $t = 0.75$

(d) Transmission velocity $r = 3$ and time $t = 1$

Fig. 6 Different pulse shape for $r = 3$ and $t = 0.25, 0.5, 0.75, 1$.

4   Findings

A comprehensive numeral simulation of the Telegraph equation based on the variations in the corresponding parameters resulted in some momentous outcomes. The damping coefficients reflect the pulse height up and down with their gradual shifting points. The time effects for stretching and compression of the pulses from left and right and vice versa for the regular shape of the pulse with lower transmission velocity. The effect of transmission velocity causes the pulses to regular for their smaller values and a quite irregular shape with the increasing values and finally, a scattered occurring while the transmission velocity reaches $r = 3$. For $r > 3$, a very scattered and complete irregular shape of the pulse heights will be forthcoming. Of all the reflections played over the pulse height, the effect of transmission velocity is quite remarkable and sounding.

5   Comparative Discussion

Since we comprised the current work with an unorthodox vision, it is arduous to compare it with the other works. Most of the works referenced in the introductory section are analytical or semi-analytical, some of the works include numerical simulations with so-called numeric calculations and manual or user-defined sketching techniques. In contrast, the approach implemented in this work is completely machine-based simulations incorporating automated graphing integrity with a programmable interface, which is computationally efficient. As we have aimed to investigate the impact of parametric variations on the pulse height (associated voltage), no solution function of explicit evaluation is required. So, the numerical approach embedded in this work is efficient for time-saving ability, which can be applied with the minimum memory requirement of the computing machine. Moreover, for different initial and boundary conditions, analytical methods need to be restarted, on the contrary, Comsol Multiphysics-based numerical simulations need to change the conditions expediently. So, the Telegraph equations with volatile features can be efficiently analyzed with the numerical simulations discussed in this work.

6   Conclusion

In this paper, we have investigated the parametric behavior of the Telegraph equation. The numerical solution has been manipulated using COMSOL Multiphysics. We analyzed a Telegraph wire of a small section and studied the pulse of voltage while moving along it. We used the parametric sweep method that gives results and shows the shape of the pulse for varying damping coefficients over several discrete time slots. We observed the changes occurring in the variation of transmission velocity. We allowed the damping coefficients to vary and kept the transmission velocity remain fixed for some specific cases then gradually increased the time slots. Then we allowed the transmission velocity to increase and kept it fixed as before and allowed the parameters p and q increasing and observed the changes at different time slots. Although the damping coefficients increased at several discrete time slots, the pulse height was smooth, symmetric, and regular for the smaller transmission velocity, for instance, $r = 1$ and $r = 2$. But the greater transmission velocity caused the pulse height to be irregular and highly scattered. A dramatic change was observed in the pulse shape for $r = 3$ which was throughout the transmission line. It is observed that pulse height is smooth, with a regular up and down for the smaller values of r irrespective of

the enhancement of the damping coefficients at the different time slots. The scattered shape of the pulse height appears as the transmission velocity increases irrespective of the enhancement of the damping coefficients at the different time slots. We hope the parametric behavioral observations are intended to be used in our further research.

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