SciEn
Journal of Engineering Advancements Vol. 02(04) 2021, pp 217-220 https://doi.org/10.38032/jea.2021.04.007

Volume Charge Density in Geometric Product Lorentz Transformation

Md. Ashraful Alam[*1] *and Atikur Rahman Baizid*[2]

[1]Department of Mechanical Engineering, University of Creative Technology, Chittagong, Chattogram-4212, Bangladesh
[2]Department of Related Subjects, Barisal Engineering College, Barishal-8200, Bangladesh

Received: October 03, 2021, Revised: December 21, 2021, Accepted: December 23, 2021, Available Online: December 26, 2021

ABSTRACT

Lorentz Transformation is the relationship between two different coordinate frames time and space when one inertial reference frame is relative to another inertial reference frame with traveling at relative speed. In this paper, we have derived the transformation formula for the volume charge density in Geometric Product Lorentz Transformation. The changes of volume charge density of moving frame in terms of that rest frame in Geometric Product Lorentz Transformation at various velocities and angles were studied as well.

Keywords: Lorentz Transformation (LT), Special Lorentz Transformation (SLT), Geometric Product Lorentz Transformation (GPLT), Volume Charge Density (VCD).

1 Introduction

In most practice on special relativity [1], a straight line motion is along with x-axis. In this case (y, z) coordinate system are remained unchangeable under the Lorentz Transformations. However, in different cases line motion does not occur simultaneously with any of the coordinate axes. Volume charge density means amount of electric charge is existing in a certain volume [2]. Bhuiyan and Baizid [3] studied equation of transformation for surface charge density in different types of Lorentz Transformations. A static cube is been used to figure out the volume of charge density in Geometric Product Lorentz Transformation. Length is contracted in Lorentz Transformations. So, volume charge density will be different in different types of Lorentz Transformations. The objective of this article is to derive the transformation equations and volume charge density of a moving frame with reference to a static frame in Geometric Product Lorentz Transformation for various velocities and angles.

1.1 Special Lorentz Transformation (SLT)

Let S and S' be two inertial frame of references, here frame S is at rest and frame S' is moving through the velocity $\vec{v}$ along x-axis with reference to the S frame. The time and space coordinates of S' and S are (x', y', z', t') and (x, y, z, t) respectively. SLT [1]-[9] which is said to be the relation between S and S', can be obtained as follows

$$x' = \gamma(x - \vec{v}t) \tag{1}$$

$$y' = y \tag{2}$$

$$z' = z \tag{3}$$

$$t' = \gamma(t - \vec{v}x) \tag{4}$$

wherein to $\gamma = (1 - \frac{v^2}{c^2})^{-\frac{1}{2}}$and $c = 1$.

In addition to the inverse of SLT [1]-[9] can be obtained as

$$x = \gamma(x' + \vec{v}t') \tag{5}$$

$$y = y' \tag{6}$$

$$z = z' \tag{7}$$

$$t = \gamma(t' + \vec{v}x') \tag{8}$$

Wherein to $\gamma = (1 - \frac{v^2}{c^2})^{-\frac{1}{2}}$ and $c = 1$.

1.2 Most General Lorentz Transformation (MGLT)

As the velocity $\vec{v}$ of S' with reference to S is not aligned with x-axis, in such a situation velocity $\vec{V}$ has three components V_x , V_y , V_z then MGLT $[2-9]$, which is the relation between the coordinates of S' and S, can be obtained as follows

$$\vec{x'} = \vec{x} + \vec{V}\left[\left\{\frac{\vec{x}.\vec{V}}{V^2}\right\}(\gamma - 1) - t\gamma\right] \tag{9}$$

$$t' = \gamma(t - \vec{x}.\vec{V}). \tag{10}$$

Also the inverse of MGLT [2]-[9] can be obtained as

$$\vec{x} = \vec{x'} + \vec{V}\left[\left\{\frac{\vec{x'}.\vec{V}}{V'^2}\right\}(\gamma - 1) + t'\gamma\right] \tag{11}$$

$$t = \gamma(t' + \vec{x'}.\vec{V}) \tag{12}$$

wherein to $\gamma = \frac{1}{\sqrt{1-\frac{V^2}{c^2}}}$ and $c = 1$.

1.3 Geometric Product Lorentz Transformation (GPLT)

Let us consider the velocity $\vec{v}$ of S' frame with reference to S frame. So the velocity $\vec{v}$ has three components v_x, v_y, v_z as the MGLT. Let $\vec{z}$ and $\vec{z'}$ be the space part in S and S' frame respectively. According to the two vectors geometric product [3],[9]-[11], can be written as $\vec{A}\vec{B} = \vec{A}.\vec{B} + \vec{A} \times \vec{B}$ here $\vec{A}$ and $\vec{B}$ are two vectors and symbol $\times$ is used as a replacement for the symbol $\wedge$.

The GPLT [3],[9]-[11] can be obtained as

$$\vec{z'} = \gamma\{\vec{z} - t\vec{v} - (\vec{z} \times \vec{v})\} \tag{13}$$

$$t' = \gamma(t - \vec{z}.\vec{v}). \quad (14)$$

Also the inverse of GPLT [3],[9]-[11] can be obtained as

$$\vec{z} = \gamma\{\vec{z'} + t'\vec{v} + (\vec{z'} \times \vec{v})\} \quad (15)$$

$$t = \gamma(t' + \vec{z'}.\vec{v}) \quad (16)$$

where about $z'_x = x'$, $z'_y = y'$, $z'_z = z'$, $z_x = x$, $z_y = y$, $z_z = z$. Since $\vec{v}$ has three components v_x, v_y, and v_z. Hence the above transformations can also be obtained as

$$x' = \gamma\{x - tv_x - (yv_z - zv_y)\} \quad (17)$$

$$y' = \gamma\{y - tv_y - (zv_x - xv_z)\} \quad (18)$$

$$z' = \gamma\{z - tv_z - (xv_y - yv_x)\} \quad (19)$$

$$t' = \gamma\{t - xv_x - yv_y - zv_z\} \quad (20)$$

and

$$x = \gamma\{x' + t'v_x + (y'v_z - z'v_y)\} \quad (21)$$

$$y = \gamma\{y' + t'v_y + (z'v_x - x'v_z)\} \quad (22)$$

$$z = \gamma\{z' + t'v_z + (x'v_y - y'v_x)\} \quad (23)$$

$$t = \gamma\{t' + x'v_x + y'v_y + z'v_z\}. \quad (24)$$

2 Methods

We describe SLT and MGLT in section 1.1 and 1.2 respectively. Section 1.3 represents the space and time for moving frame and rest frame of Geometric Product Lorentz Transformation. We use volume charge density of section 3 to obtain transformation equation for VCD of moving frame in terms of that of rest frame in Geometric Product Lorentz Transformation in section 4.

3 Volume Charge Density (VCD)

The total amount of electric charge in one unit volume is said to be VCD which is designated by σ, where $\sigma = \frac{q}{V}$. It is calculated as coulombs per cubic meter $\left(\frac{c}{m^3}\right)$. VCD can take on negative values. Elementary charge e $(=1.6 \times 10^{-19} coul)$ is formed in the minimum charge of proton or electron. It may decide that the electric charge is relativistically unchangeable because overall elementary charge is independent on the condition of the motion towards the viewer. On the basis of this discussion we can derive equation of transformation for VCD σ [3],[8].

4 Transformation of Geometric Product Lorentz Transformations

Let's consider two inertial frame S and S' where S' is moving through uniform velocity $\vec{v}$ respect to the frame S along any arbitrary direction which is shown in Fig. 1. Therefore the velocity $\vec{v}$ has three components v_x, v_y, and v_z. Let's examine a uniform static cube having charge density $+\sigma$ coul/m^3 at steady in frame S which one side is parallel to x-axis. Let's choose the cube edge length is R_0. The spectator in S' will detect that the cube is passing X-Y plane in opposed direction having velocity $\vec{v}$ towards any random direction.

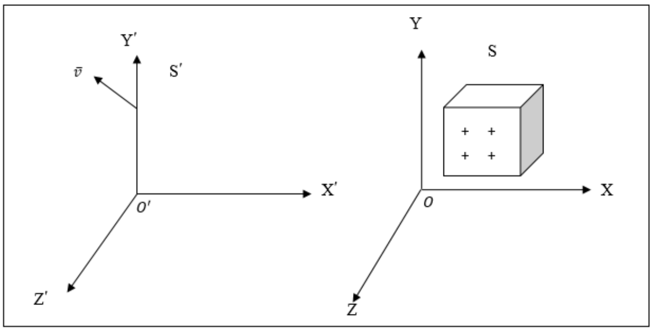

Fig. 1 X-Y plane having velocity $\vec{v}$ in the S'frame with respect to the S frame.

If R_0 is in S frame, then length of contraction for the GPLT in S' frame can be written as [12]

$$\vec{R_0} = \gamma\{\vec{R} - (\vec{R} \times \vec{v})\}.$$

$$\text{Or, } {R_0}^2 = [\gamma\{\vec{R} - (\vec{R} \times \vec{v})\}]^2.$$

$$\text{Or, } {R_0}^2 = \gamma^2\{R^2 - (\vec{R} \times \vec{v})^2 - \vec{R}(\vec{R} \times \vec{v}) - (\vec{R} \times \vec{v}).\vec{R} + (\vec{R} \times \vec{v}) \times (\vec{R} \times \vec{v})\}$$

$$= \gamma^2\{R^2 + (Rv\sin\theta)^2\}.$$

$$\text{Or, } {R_0}^2 = \gamma^2\{R^2 + R^2v^2(1 - \cos^2\theta)\}.$$

Considering R has three components R_x, R_y, and R_z.

$$\text{Or, } R^2 = \frac{{R_0}^2}{\gamma^2\{1+v^2(1-\cos^2\theta)\}}.$$

$$\text{Or, } {R_x}^2 + {R_y}^2 + {R_z}^2 = \frac{{R_{0x}}^2 + {R_{0y}}^2 + {R_{0z}}^2}{\gamma^2\{1+v^2(1-\cos^2\theta)\}}.$$

$${R_x}^2 = \frac{{R_{0x}}^2}{\gamma^2\{1+v^2(1-\cos^2\theta)\}}.$$

$${R_y}^2 = \frac{{R_{0y}}^2}{\gamma^2\{1+v^2(1-\cos^2\theta)\}}.$$

$${R_z}^2 = \frac{{R_{0z}}^2}{\gamma^2\{1+v^2(1-\cos^2\theta)\}}.$$

Therefore total charge detected by a spectator in the S' frame is

$$Q' = R_xR_yR_z\sigma'.$$

$$\text{Or,} Q' = \left(\frac{{R_{0x}}^2}{\gamma^2\{1+v^2(1-\cos^2\theta)\}}\right)^{\frac{1}{2}} \times \left(\frac{{R_{0y}}^2}{\gamma^2\{1+v^2(1-\cos^2\theta)\}}\right)^{\frac{1}{2}} \times \left(\frac{{R_{0z}}^2}{\gamma^2\{1\mp(1-\cos^2\theta)\}}\right)^{\frac{1}{2}}\sigma'.$$

$$\text{Or, } Q' = \frac{{R_0}^3}{[\gamma^2\{1+v^2(1-\cos^2\theta)\}]^{\frac{3}{2}}}\sigma'.$$

Following the law of conservation of charge $Q' = Q_0$

$$\text{Or, } \frac{{R_0}^3}{[\gamma^2\{1+v^2(1-\cos^2\theta)\}]^{\frac{3}{2}}}\sigma' = {R_0}^3\sigma.$$

$$\text{Or, } \sigma' = \sigma[\gamma^2\{1 + v^2(1 - \cos^2\theta)\}]^{\frac{3}{2}}.$$

This is the equation of transformation for volume charge density in Geometric Product Lorentz Transformations.

5 Results and Discussion

Table 1 Space, time, and transformation equation for volume charge density in Geometric Product Lorentz Transformations.

Length of Contraction	$\overrightarrow{R_0} = \gamma\{\vec{R} - (\vec{R} \times \vec{v})\}$
Volume Charge Density	$\sigma' = \sigma[\gamma^2\{1 + v^2(1 - cos^2\theta)\}]^{\frac{3}{2}}$
Space	$x' = \gamma\{x - tv_x - (yv_z - zv_y)\}$ $y' = \gamma\{y - tv_y - (zv_x - xv_z)\}$ $z' = \gamma\{z - tv_z - (xv_y - yv_x)\}$
Time	$t = \gamma\{t' + x'v_x + y'v_y + z'v_z\}$

5.1 Graphical Depiction of VCD in GPLT

Volume charge density in moving frame in terms of rest frame indicates black color, red color, and blue color straight line for respective volume 0.2 c, 0.4 c, and 0.6 c in Fig. 2-Fig. 5 where speed of light, c = 300,000 km/sec.

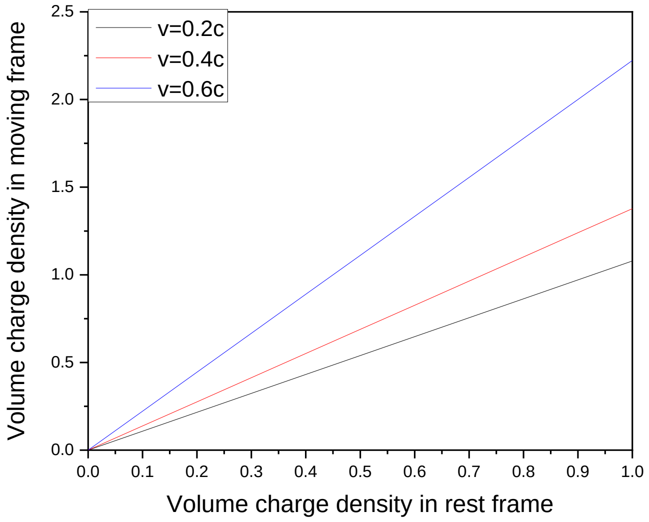

Fig. 2 VCD in GPLT at 30 degree angle

In Fig. 2, VCD in moving frame increases with volume rising for same 30 degree angle. It is clear that black color straight line below than all other straight line and blue color straight line upwarded among all. Moreover, we can see that only black color straight line is observing strictly at 30 degree angle.

In Fig. 3, VCD in moving frame rises with volume increasing for same 45 degree angle. It is notable that black color straight line under than all other straight line and blue color straight line upstairs among all. Furthermore, we can realize that only blue color straight line is observing firmly at 45 degree angle.

In Fig. 4, VCD in moving frame at 60 degree angle performs as like as for 45 degree angle. Additionally, we can observe that only blue color straight line is observing conclusively at 60 degree angle.

In Fig. 5, VCD in moving frame at 75 degree angle acts as like as for 30 degree angle. Additionally, we can notice that no color straight line is observing decisively at 75 degree angle.

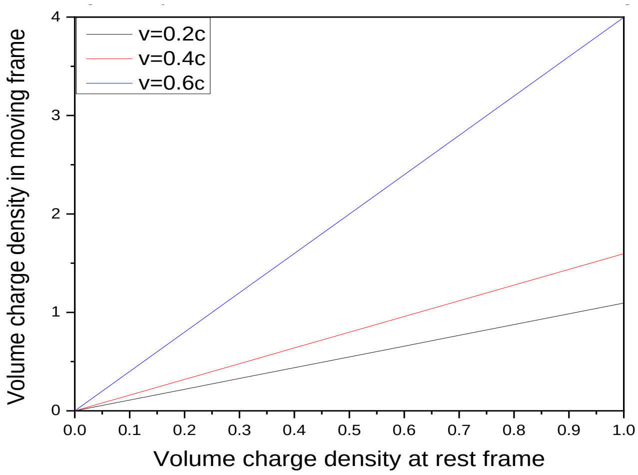

Fig. 3 VCD in GPLT at 45 degree angle

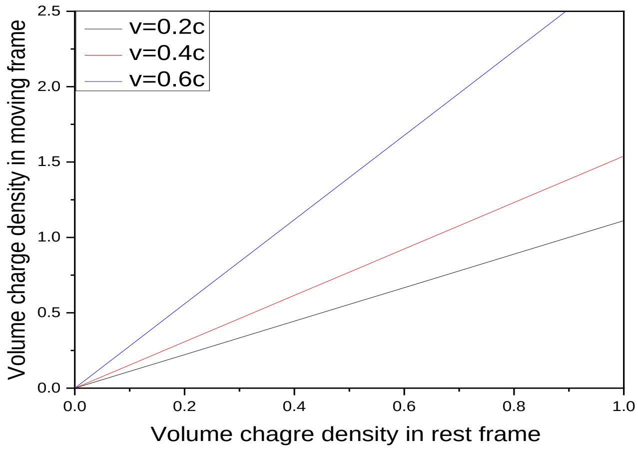

Fig. 4 VCD in GPLT at 60 degree angle

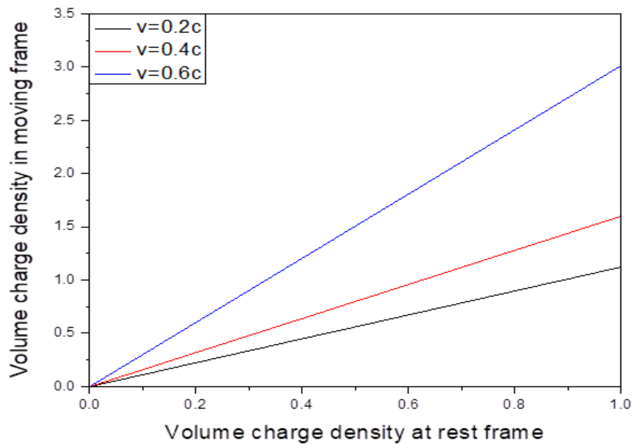

Fig. 5 VCD in GPLT at 75 degree angle

6 Conclusion

In special relativity, space-time is mainly four dimensional and time cannot be disconnected from three dimensional spaces. The space-time and transformation equation for volume charge density in Geometric Product Lorentz Transformation is displayed in Table 1. Volume charge density based numerical values of Geometric Product Lorentz Transformation for a moving frame with reference to a static frame have been carried out and graphically it is observed that for same angles the value of the VCD for a moving frame increases with the rise of velocity of the moving frame itself.

References

[1] Resnick, R., 1994. *Introduction to Special Relativity*. John Wiley & Sons, USA, ISBN: 978-0-471-71725-6.

[2] Bhuiyan, S.A. and Baizid, A.R., 2016. Volume Charge Density in Most General Lorentz Transformation. *Journal of Scientific Research*, *8*(3), pp.259-265.

[3] Bhuiyan, S.A. and Baizid, A.R., 2015. Surface Charge Density in Different Types of Lorentz Transformations. *Applied Mathematics*, *5*(3), pp.57-67.

[4] Bhuiyan, S.A., 2019. Volume Charge Density in Mixed Number Lorentz Transformation. *Journal of Scientific Research*, *11*(2), pp.209-214.

[5] Moller, C., 1972. *The Theory of Relativity*. Oxford University Press, London, UK.

[6] Prakash, S., 1993-1994. *Relativistic Mechanics*. Pragati Prakashan.

[7] Baizid, A.R. and Alam, M.S., 2012. Applications of different Types of Lorenz Transformations. *American Journal of Mathematics and Statistics*, *2*(5), pp.153-163.

[8] Rafiq, S.B. and Alam, M.S., 2012. Transformation of Surface Charge Density in Mixed Number Lorentz Transformation. *Sri Lankan Journal of Physics*, *13*(1), pp.17-25.

[9] Baizid, A. and Alam, M., 2014. Reciprocal Property of Different Types of Lorentz Transformations. *International Journal of Reciprocal Symmetry and Theoretical Physics*, *1*(1), pp.19-35.

[10] Datta, B.K., De Sabbata, V. and Ronchetti, L., 1998. Quantization of gravity in real space-time. *Nuovo Cimento. B*, *113*(6), pp.711-732.

[11] Datta, B.K., Datta, R. and De Sabbata, V., 1998. Einstein field equations in spinor formalism: a clifford-algebra approach. *Foundations of Physics letters*, *11*(1), pp.83-93.

[12] Alam, M.S., Begum, K., 2009. Different Types of Lorentz Transformations, *Jahangirnagar Phy. Stud.* 15.